THE INFLUENCE OF REINFORCING STEEL BARS ON THE APPARENT LONGITUDINAL ULTRASONIC PULSE VELOCITY IN CONCRETE: A SIMPLE MATHEMATICAL MODEL.

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Low ultrasonic frequencies must be used in non-destructive testing of heterogeneous materials, such as plain and reinforced concrete, cast iron or timber. When frequencies are low enough, size and shape of tested bodies may influence measured longitudinal pulse velocities.
An approach which is named "quasi-fluid" is applied to discuss the longitudinal pulse propagation in reinforcing steel bars embedded in concrete. Wave propagation is described by a spatially-averaged dilatational field.
An approximate formula is obtained which relates group velocity with an effective radius of the embedded bars, with transducer frequency, and with the asymptotic P-wave velocity in steel and concrete, amongst other parameters. The evolution of a typical ultrasonic pulse is described using stationary-phase methods.
Several known experimental facts are thus explained.
Finally, common-use empirical correlations between measured longitudinal pulse velocity and the diameter of embedded steel bars are discussed using the model developed in this paper.

Key words: ultrasounds, wave propagation, non destructive testing, concrete.

1. INTRODUCTION

Ultrasonic testing of concrete is currently used for quality assessment in large concrete structures cast on site and in mass-produced prefabricated units.
The intrinsic heterogeneity of concrete limits the carrier frequencies that can be used to 250 kHz and less (being 54 kHz the most common frequency).
Besides, it usually demands two ultrasonic probes (transducers), one for pulse emission and the other for pulse reception. In practice, probes for ultrasonic testing of concrete are not much larger (50 mm) than the customary ones for metals. The contact face of a common cylindrical probe, whose diameter is of the order of magnitude of the wavelength, radiates besides longitudinal waves, also shear and surface waves of appreciable intensity.
The absence of any directional effect in the probes, as well as the multiple scattering of the waves inside concrete, makes it possible, (in principle) to couple directly any two points on the surface of the specimen being tested.
The fastest ultrasonic signal received in this way is then always a direct longitudinal wave-pulse. This is followed by shear and surface wave-pulses, and by reflected longitudinal waves. For quality tests on concrete the longitudinal acoustic velocity is determined measuring the distance between probes and the time that the direct longitudinal pulse takes to go from the emitting to the receiving probe. A higher

velocity usually means a better concrete quality (strength, durability and dimensional stability).

Reinforcement, if present, should be avoided. Indeed, considerably uncertainty is introduced by the higher velocity of pulses in steel and by possible compaction shortcomings in heavily reinforced regions. If the reinforcing bars run in a direction at right angles to the pulse path and the sum of bar diameters is small in relation to path length, the effect is generally not significant. But if the reinforcing bars lie along or parallel to the path of the pulse, the effect may be very significant. It is not only the measurements taken along the reinforcing bars that are affected, but also those in the neighbourhood of the bars. The first pulse taking and indirect path via the reinforcing bar may reach the receiving probe before that going along the direct path through concrete, resulting in a shorter apparent transit time. Now, a relatively small difference in pulse velocity usually corresponds to a relatively large difference in the quality of concrete. So, if reinforcement cannot be avoided, it may be necessary to make some corrections in order to estimate the true longitudinal pulse velocity in bulk concrete.

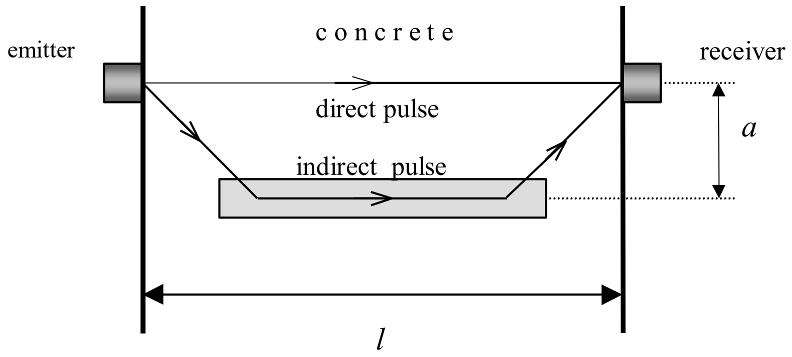

Fig. 1

In BS 4408 pt5, the following correction is recommended for the situation depicted in fig 1.(Bungey,1982).If t is the measured travel time of the pulse, l the apparent path length, V_S is the pulse velocity in steel, and a is the distance shown in the figure, then the true pulse velocity in bulk concrete, V_c, is given by

$$V_c = \frac{2aV_s}{\sqrt{4a^2 + \left(tV_s - l\right)^2}} \qquad [1]$$

The apparent pulse velocity in bulk concrete would be

$$V_{c,app} \equiv \frac{l}{t} \qquad [2]$$

The correction may be applied if $V_c \leq V_s$ and if

$$\frac{a}{l} \leq \frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s - V_c}} \qquad [3]$$

The effect of the reinforcement disappears if

$$\frac{a}{l} > \frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s + V_c}} \qquad [4]$$

The aforementioned correction formula is based in the acoustic ray theory of the old refraction method of the seismologists (Pilant, 1979).In this case we have an elastic layer of thickness a over an elastic half-infinite bed. P waves are produced by an emitter at the upper face of the layer, and the waves are detected by a receiver, also located at the upper face (Fig.2).

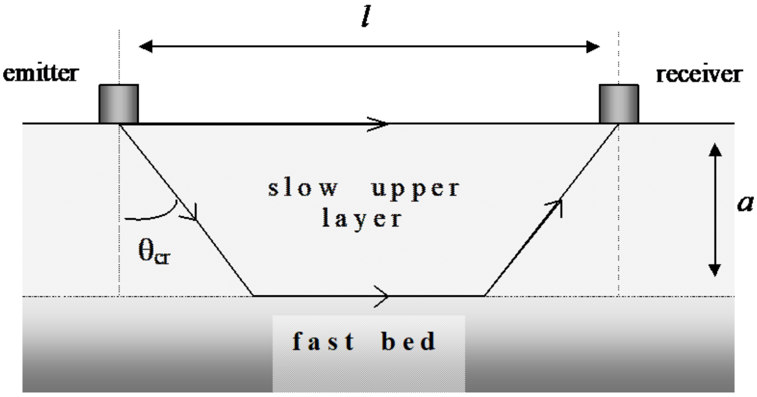

Fig. 2

If the bulk P wave velocity in the bed (V_{L2}) is higher than the bulk P wave velocity in the layer (V_{L1}), the method may be applied to determine a, V_{L1} and V_{L2} varying l and measuring the corresponding travel times t. If $\frac{a}{l}$ is greater than the threshold value

$$\frac{1}{2}\sqrt{\frac{V_{L2} - V_{L1}}{V_{L2} + V_{L1}}} \qquad [5]$$

the direct P wave arrives first. But if $\frac{a}{l}$ is less than this threshold, an indirect P wave arrives first. This wave is totally reflected at the interface. (All the other indirect P waves arrive later). If the layer is a layer of concrete and the bed is a steel bed, V_{P1} would be V_c and V_{P2} would be V_S= 5.90 km/sec. However, Chung (1978) has shown that for pulses travelling in the direction of the axis of the reinforcing bars through a steel-concrete medium, the effective pulse velocity V_e is less than the above mentioned value for P waves in bulk steel and it is influenced by bar diameter according to the approximate empirical formula:

$$V_c = 5.90 - \frac{5.2}{R}.(5.90 - V_c) \qquad [6]$$

Here R is bar radius (in mm) and V_c is the true velocity in concrete (in km/sec.)

(The radius R must be greater than 5.2 mm. Chung´s formula was obtained for pulses of 54 kHz).

As a consequence, it seems that V_e must be used instead of V_S in formula (1), giving an implicit relation for V_c. However, even now there is controversy about the meaning and the feasibility of these corrections. The first critical discussion seems to be that made by Bungey, (1984).

The purpose of the present work is to discuss some aspects of the longitudinal pulse propagation in reinforcing steel bars, with the help of a fairly simple mathematical model.

Even if enough spatial symmetry is assumed, the problem of pulse propagation in steel bars embedded in concrete is fairly difficult to solve using directly an ab-initio numerical analysis of the equations of elasto-dynamics (Bellomo and Preziosi, 1995; Conca and Gatica, 1997). So, a simplified analytical approach, as intended in this paper, may be of interest, both for the design of complex digital simulations and for the interpretation of the experimental results obtained with non-destructive testing (ultrasonic) equipments.

2. THE ALMOST-FLUID APPROACH TO LONGITUDINAL PULSE VELOCITY AND THE EQUIVALENT LEAKY WAVEGUIDE.

Let us consider first a steel bar in air. It is a solid wave guide with free (zero-stress) boundary conditions. From the equations of elasto-dynamics written in cylindrical coordinates (Pochhammer´s equations), and assuming enough symmetry to exclude solutions that correspond to flexural or to torsional vibrations (Kolsky,1964), a numerable infinity of longitudinal propagation modes is obtained (for an infinite waveguide). Each mode is given by a definite dispersion relation between angular frequency ω and wave number k along the waveguide axis, and by a certain modal field of vector displacements in any cross section of the bar. This field is a function of k, and k is an increasing function of ω.

With the exception of the lowest order mode, all the others show a low-frequency cut-off. If ω is below mode´s cut-off frequency, k is imaginary and the mode is non-propagating. When k approaches zero, the phase velocity ω/k for the lowest mode approaches the extensional wave velocity, while the higher modes are non-propagating. When k grows without bound the phase velocity of the lowest mode tends to the velocity of Rayleigh´s surface waves and the corresponding displacement field concentrates in a thin layer adjacent to the boundary of the bar (skin effect).When k grows, the phase velocity of the next higher modes tends to the shear wave velocity. The phase velocity of the highest order modes tends to the dilatational (P) wave velocity.

All this stemmed from numerical analysis of Pochhammer´s analytic solutions to the equations of elasto-dynamics (Auld, 1973).

Now, the conditions at the emitting probe are never such us to initiate precisely the waves indicated by the Pochhammer solutions. As the wave train progresses along the bar, disturbances along the boundary are produced by reflection and mode conversion so as to reduce there the stress to zero. As a consequence, at the receiving probe a complex time signal is obtained corresponding to a longitudinal wave packet arriving first followed by several trailing pulses. The experimental results suggest that, as a first approximation when kR is a big number, the dilatational component may be considered to be guided along the bar much as in the case of a fluid wave-guide that releases pressure at its boundary (Mc Skimin,1956).But in the solid wave guide case, energy is continually being drained off through mode conversion processes at the boundary. Then, at any cross section of the bar the leading pulse can be build up from the Fourier spectrum of the emitted pulse using only the dilatational part of the Pochhammer solutions.

This is the almost-fluid approximation, which is very useful to consider the effects of the geometric dispersion when the wavelengths are small relative to the smallest dimension of the solid body perpendicular to the direction of pulse propagation (Suárez Antola,1990).The displacement field imposed by the emitting transducer excites mainly the first mode of the almost fluid waveguide, whose group velocity is given by

$$V_g = V_L \sqrt{1 - \frac{\omega_{MS}^{\;2}}{\omega^2}} \qquad [7]$$

Here V_L is P-wave velocity in steel and

$$\omega_{MS} = \frac{2.40 * V_L}{R} \qquad [8]$$

is a cut-off frequency. We propose to call it "Mc Skimin frequency".(2.40 is the first zero of $J_o(x)$, the Bessel function of the first kind and order zero). This formula can be applied when kR is much greater than one, that is, for high frequencies. But for the low frequencies used in ultrasonic testing of concrete the wavelengths $\lambda = 2\pi/k$ are not small in comparison with bar radius R.(for 54 kHz in steel, $\lambda = 0.11$ m). So, even if the spectra of short ultrasonic pulses of 54 kHz of carrier frequency have significant components at higher frequencies (above 150 kHz), it is not at all clear whether we can study the travel time of the leading dilatational pulse using the almost-fluid approach.

Furthermore, our objectives are in steel bars embedded in concrete. As V_L' (P wave velocity in concrete) and V_T' (S wave velocity in concrete) are less than V_L and V_T in steel, there is no possibility of total reflection when a wave comes to the steel-concrete interface from the steel side. Then a dilatational pulse that travels along an embedded reinforcement bar is continuously loosing energy towards the adjacent concrete. Once the waves go through the steel-concrete interface towards the concrete, they scatter, reflect and convert modes many times at the interfaces between the aggregates and the cement paste. Part of this energy comes back and now part of it can be trapped by total reflection in the concrete layer adjacent to the steel bar. A certain dilatational wave field is thus produced, surrounding the bar and inside the bar, moving along the bar axis and tending fairly quickly to zero as the distance from the bar axis grows. Our suggestion now is to model the propagation of the leading longitudinal pulse (whose travel time is measured in non-destructive testing) using an equivalent leaky waveguide, coaxial with the steel bar, but with an effective radius R_e, greater than bar radius R. This equivalent waveguide will be in principle a solid one. However, to simplify our analysis, let us suppose for a moment that it can be treated as almost-fluid. Then, for the first propagation mode we can apply formula [7] for the group velocity, but with R_e instead of R in the equation [8] for Mc Skimin frequency. The radial dependence of the dilatational field for this mode is given by $J_o(2.40r / R_e)$, where r is the distance from bar´s axis. Now, we can write approximately: $J_o(x) = 1 - (x^2/(2.40)^2)$.

Introducing the parameter

$$\theta = J_0\left(2.40\frac{R}{R_e}\right) \qquad [9]$$

we have

$$R_e \approx \frac{R}{\sqrt{1-\theta}} \qquad [10]$$

In general, besides the wave field that propagates through the equivalent leaky waveguide, there is a direct longitudinal pulse that travels with velocity V_L' through bulk concrete. When the radius of the bar is (see appendix):

$$R_0 = \frac{2.40\sqrt{1-\theta}.V_L}{\omega.\sqrt{1-\left(\frac{V_L'}{V_L}\right)^2}} \qquad [11]$$

we have $V_g = V_L'$. If R is greater than R_o, the first pulse that arrives to the receiving transducer would be the leaky waveguide pulse. If R is less than R_o, it would be the direct pulse that travels with velocity V_L'. Then, the effective velocity V_e of longitudinal pulses in reinforced concrete would be given by equation [7] with R_e instead of R, if $R_0 \leq R < +\infty$, and by $V_e = V_L$' if $R \leq R_0$ .But the formula for group velocity in the leaky waveguide can be fairly well approximated by

$$V_e \approx V_L - (V_L - V_L')\frac{R_0}{R} \qquad [12]$$

Both give the value V_L' when $R=R_o$, both grow monotonically with R in a hyperbolic fashion and both tend to V_L when R tend to infinity.

Formula [12] jointly with formula [11], constitute a theoretical counterpart of Chung′s empirical formula (equation [6]). Equation [6] was obtained adjusting parameters to the measurements taken from concretes with six different values of the bulk velocity V_L'.

As the empirical value for R_o does not seems to depend of V_L', from the theoretical formula for R_o (equation [11]) we find that $\theta \approx \left(\frac{V_L'}{V_L}\right)^2$. Then

$$R_0 \approx \frac{2.40 * V_L}{\omega} \qquad [13]$$

and

$$R_e = \frac{R}{\sqrt{1-\left(\frac{V_L'}{V_L}\right)^2}} \qquad [14]$$

If the steel bar is in void ($V_L' = 0$), we obtain R_e=R as it should be. If V_L' approaches to V_L, R_e tends to infinity. This means that J_0 (2.40r / R_e) will tend to one for every r and so we will have homogeneous plane waves moving without geometric dispersion, also as it should be.

Now let us put a transducer to measure, at a certain cross-section of the waveguide, a propagating wave packet. The probe produces a scalar (voltage) time signal after making a certain weighted average of the elasto-dynamic field. So, it will be enough to consider the propagation of a representative scalar field (for example, the average displacement) given by:

$$u(t,z) = \int_{-\infty}^{+\infty} A(\omega).e^{i\phi(\omega)}.e^{-z\alpha(\omega)}.e^{i(\omega t - z.k(\omega))}d\omega \qquad [15]$$

Here α(ω) is an attenuation coefficient related with the dilatational energy drainage off the equivalent waveguide, by mode conversion and by leakages towards the bulk of concrete.(We are not considering the energy dissipation due to internal friction or coupled elastic and thermal effects). By analogy with the bar in air case (Suárez Antola, 1998), it is possible to guess that

$$\alpha(\omega) \approx \frac{V_L.g\left(\frac{R_e.\omega}{V_L}\right)}{\omega.R_e^2} \qquad [16]$$

Here g(x) is a slowly varying function of its argument. When t is big enough, the method of stationary phase (Segel, 1987; Pilant, 1979) can be applied to the evaluation of the integral [15]. For a given t and z, the main contribution to the integral comes from the frequencies $\omega = \omega_e(t, z)$ that verify

$$\frac{\partial k}{\partial \omega}(\omega_e) = \frac{t}{z} \qquad [17]$$

.

If [17] does not have real roots, u (t, z) is negligible. In our case, from [7] and [17] it follows that

$$\omega_e = \frac{\omega_{MS}}{\sqrt{1-\left(\frac{z}{V_L.t}\right)^2}} \qquad [18]$$

so that we have a single real root if z is less than V_Lt. (For the equivalent leaky waveguide, R_e must be put instead of R in the expression [8] for ω_{MS}).

The scalar field [15] can be approximated by (Segel, 1987; Pilant, 1979):

$$u(t,z) \approx \frac{2e^{-z.\alpha(\omega)}.A(\omega_c)}{\left[\frac{1}{2\pi}.\left|\frac{\partial^2 k(\omega_c)}{\partial \omega^2}\right|.z\right]^{\frac{1}{2}}}.\cos\left(\omega_e.t - k(\omega_e).z + \phi(\omega_e) + s.\frac{\pi}{4}\right) \qquad [19]$$

Here s is the sign of $\frac{\partial^2 k}{\partial \omega^2}(\omega_e) \neq 0$.

Formula [19] is a local approximation by an harmonic wave of frequency ω_e and wave number $k(\omega_e)$.

But when z/t varies, ω_e changes as well (according to formula [18]),so that we obtain a wave modulated in amplitude, frequency and phase.

The amplitude spectrum A (ω) depends of the emitted pulse. Consider an incoming pulse

$u(t,0) = A_o e^{-\mu t}(\sin\omega_0 t)H(t)$ (ω_0 is the carrier frequency and H(t) is Heavside´s unit step function).Then we have

$$A(\omega) = \frac{\omega_0}{\sqrt{\left(\mu^2 + \omega_0^2 - \omega^2\right)^2 + 4\mu^2.\omega^2}} \qquad [20]$$

If the pulse is a short one, as in ultrasonic non-destructive testing, we can put $\mu \approx \omega_0/2\pi$. So, even if we have spectral components of every possible frequency, their amplitudes tend to zero as $1/\omega^2$, when ω grows to infinity. Let $\omega_{e,u}$ be the biggest frequency whose amplitude is over the detection threshold. Then from equation [17] it follows that for a certain position z of the receiving transducer, the first signal detected would be registered in an instant t such that

$$\frac{(z/t)}{V_L} = \sqrt{1 - \frac{\omega_{MS}^2}{\omega_{e,u}^2}} \qquad [21]$$

This would give us the apparent dilatational pulse velocity, measured with a propagating wave-packet of a given spectrum of Fourier components.

The effective longitudinal pulse velocity V_e would be given by formula [7], but for $\omega = \omega_{e,u}$ instead of the carrier frequency ω_0 of the emitted pulse. Furthermore, we must use $\omega_{e,u}$ instead of ω_0 in formula [13] for the radius R_o of the bar such that for it $V_e = V_L'$.

3. DISCUSSION AND CONCLUSIONS.

In order to describe the propagation of the leading dilatational pulse in a reinforcement bar embedded in bulk concrete, we introduced an equivalent solid waveguide of radius R_e (given by formula [14]), greater than the geometric radius of the bar. When the number $R_e\omega / V_L$ is big enough (that is, for frequencies high enough) this solid waveguide behaves as almost fluid, in the following sense. Due to interference from waves produced at interfaces, both the axial and the radial displacement fields, as functions of the distance from bar axis, show small oscillations superposed on a fairly smooth trend. If we make an average of these fields over a suitable chosen (and movable) plane region, we obtain a smoothed axial field that seems to obey very approximately a leaky classical wave equation, with velocity V_L and supported by a (fluid) waveguide of radius R_e .If we neglect the influence of the leaks on the wave velocity, we obtain the results stated in section (2).But if $R_e\omega / V_L$ is not big enough, this almost-fluid model cannot be applied and a more realistic model of an equivalent solid waveguide must be constructed. The simplest solid waveguide model can be obtained coupling a purely extensional mode with a suitable shear mode. This, as will be developed elsewhere, gives two emerging modes: a lower extensional-interface mode and an upper shear-dilatational mode. When $R_e\omega$ / V_L increases, this upper mode approaches the first dilatational mode of the almost fluid waveguide. At its turn, the lower mode approaches an interface wave propagation mode, which is very slow and it is less and less excited by the emitting transducer as $R_e\omega$ / V_L increases.

The leading edge of the first dilatational pulse is occupied by the highest frequencies. So we can expect to be able to discuss the measured transit time between emitter and receiver using the almost-fluid approach.

In principle, the spectrum of the incoming pulse has component frequencies as big as we want. As a consequence, a certain fraction of the pulse will always travel with a velocity very near to V_L .But this fraction can be detected, and its corresponding travel time can be measured, only if the amplitude of the cosine in formula [19] is above the detection threshold of the ultrasonic equipment. This is the case when the length l of the trajectory of the pulse is of the same order of magnitude or less than the radius R_e of the equivalent waveguide: here we does not have geometric dispersion. But when l increases, this high frequency part of the wave-packet will remain under the threshold of detection and the measured transit time will correspond to the travel time of the highest detectable frequency $\omega_{e,u}$. This threshold frequency will depend of l, of the amplitude spectrum of the emitted ultrasonic pulse, of the signal to noise characteristics of the time measuring equipment and of the propagation distortions from the emitter to the receiver described by formula [19]. The fluid waveguide model predicts that $\omega_{e,u}$ should grow if l decreases, if ω_0 (the carrier frequency) increases, and if the total energy of the incoming pulse increases (for the same relative amplitudes).So, if we insist in using the theoretical version of Chung´s formula developed in section (2), from the above remarks and from equation [13] it follows that R_o should decrease if l decreases, or if ω_0 increases, or if the total energy of the pulse increases. From Chung´s experimental value for R_o (0.0052m) and using [13], it results $\omega_{e,u} \approx 14.160/0.0052 = 2.723$ MHz. The carrier frequency is $\omega_0 = 6.28*540$ kHz = 0.3391 MHz so that in this case $\omega_{e,u}$ would be nearly eight times ω_0 .(In Chung´s experiments l was fixed at near 2m).

Using the concept of an equivalent almost-fluid waveguide we have thus explained some known aspects of the geometric dispersion of longitudinal pulses in reinforcement bars. Furthermore, we have seen that several qualitative and quantitative predictions, amenable to experimental testing, stem from the results obtained in section (2).

4. APPENDIX

The group velocity of the fluid waveguide is given by formula [7], and the Mc-Skimin frequency is given by formula [8]. When $R = R_0$, then by formula [10] we have $R_{e,0} = \dfrac{R_0}{\sqrt{1-\theta}}$. From equations [7] and [8], taking into account that when $R = R_0$, then $V_g = V'_L$, we obtain: $R_{e,0} = \dfrac{\dfrac{2.40.V_L}{\omega}}{\sqrt{1-\left(\dfrac{V'_L}{V_L}\right)^2}} = \dfrac{R_0}{\sqrt{1-\theta}}$. From this last equality formula [11] in the text follows.

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MODE COUPLING AND THE GEOMETRIC DISPERSION OF LONGITUDINAL ACOUSTIC SIGNALS IN SOLID WAVEGUIDES: AN ANALYTICAL APPROACH.

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Abstract: The effect of geometric dispersion in the propagation of wave-packets in bars appears when the main wavelengths are of the same order of magnitude that the least dimension of the cross - section of the guide. It is important to take this into account in the design of certain acoustic delay lines and in the interpretation of certain low frequency ultrasonic signals used in non-destructive testing. Three propagation modes are selected from known solutions of Pochhammer equations with mixed boundary conditions. Then, a simple mathematical model for longitudinal pulse propagation is obtained by suitably coupling the selected mode amplitudes, in order to comply with zero—stress boundary conditions. The resulting system of partial differential equations allows an elegant analytical approach to the effect of geometric dispersion in pulse propagation, using only time and one space coordinate along the axis of the bar as independent variables. Several experimental results are discussed within the framework of the proposed model. Besides, its limitations are assessed and new experiments and digital simulations are suggested to test some predictions stemming from the model.

Key words: solid acoustics; geometric dispersion; mathematical models; longitudinal pulses; mode coupling.

1. Introduction

Longitudinal ultrasonic pulses are currently used in non-destructive testing of heterogeneous materials, like concrete, rocks, timber and other composites. High ultrasonic frequencies are strongly attenuated by these materials, by absorption and scattering, so that relatively low ultrasonic frequencies must be used in most routine tests. However, when the tested body is slender and the frequencies are low enough, the size and shape of the body may influence the measured longitudinal pulse velocities (Jones, 1967; Bungey, 1982).
As an example Tab.(1) shows the relation between measured longitudinal pulse velocities along reinforcing steel bars in air, and bar radius, (for pulses with a carrying frequency of 54 kHz) (H. Chung, 1978).

Table 1. Relation between longitudinal pulse velocity and bar's radius.

bar radius (m)	pulse velocity (m/s)
3.15 E-3	5180
6.35 E-3	5350
8.50 E-3	5480
12.70 E-3	5550
19 E-3	5690
50 E-3	5840

The decrement in pulse velocity when the radius of the bar decreases is due to geometric dispersion. Phase velocity varies with frequency and with the size and shape of the cross-section, adding an unwanted complication to the ultrasonic test. However, geometric dispersion may be of interest for the design of certain acoustic delay lines.

The purpose of the present work is to develop a simple mathematical model of geometric dispersion for longitudinal pulse propagation in solid waveguides.

For an infinite cylindrical waveguide, with zero stress boundary conditions and assuming a purely sinusoidal time dependence, the equations of elasto-dynamics were posed in cylindrical coordinates and solved by separation of variables by Pochhammer as early as in 1876 (Kolsky, 1963). The boundary conditions then give a trascendental equation that relates the angular frequency ω with the wave number k along the axis of the rod: the so called dispersion relation, in terms of Bessel functions. Assuming a suitable spatial symmetry (to eliminate flexural and torsional vibrations) a numerable infinity of propagation modes is obtained from the aforementioned dispersion relation. Each mode is characterized by a definite connection between ω and k, as well as by certain typical space patterns of strain and stress. In principle, by superposition of the contributions of the different modes and the different frequencies that belong to pulse spectrum, we can follow the propagation and distortion of a given longitudinal ultrasonic pulse along the axis of a bar. Some work along these lines was done at the end of the fifties and during the sixties. It was soon realized that it may be actually easier to make an ab-initio numerical analysis without the introduction of the mode concept and without spectral decomposition of the pulse. This is the contemporary trend, fostered by the increase in computer power and the development of high quality numerical algorithms. But even with the present computing facilities, the problem of geometric dispersion can be fairly difficult to tackle in certain cases of practical interest. One example is the propagation of longitudinal ultrasonic pulses in steel bars embedded in plain concrete (Chung, 1978; Bungey, 1984; Suárez Antola and Suárez Bagnasco, 1999).

So, a simplified analytical approach may be of interest, both as a guide for the design of ab-initio digital simulations of pulse propagation and for the interpretation of experimental results obtained with non destructive testing equipments.

Let us make first a brief review of the previous work related with the construction of simple mathematical models of geometric dispersion in longitudinal waves.

2. Critical review of some aspects of the theory of Giebe and Bletchschmidt and the almost fluid waveguide.

Between 1876 and 1930 several efforts were done to identify longitudinal propagation modes in elastic bars of circular cross section, making analytical approximations in the original dispersion relation obtained first by Pochhammer and later by Chree (Rayleigh, 1894) or even posing new and simplified field equations (Love 1927). This efforts were successful only for one particular mode and when kR is much smaller than one (long wavelengths relative to the bar radius R). Soon after that, Giebe and Blechschmidt (1933) developed a highly simplified theory (GB theory) to study mechanical resonance in bars of any cross section (Kolsky, 1963). The bar was treated as two mechanical systems coupled together: an elastic bar of infinitesimal cross-section in longitudinal vibration and an elastic plate, of infinitesimal thickness in radial vibration. The coupling produced two modes. In the ω versus k plane, each mode is given by a curve (a branch of the corresponding dispersion relation) (Fig. (1)).

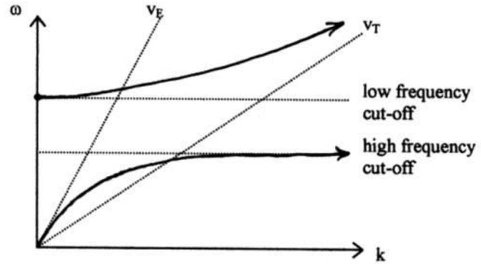

Figure1. Upper and lower modes in the Giebe and Blechschmidt theory

The slope (that is, the group velocity) of the lower branch when k is near zero (long wave-lengths) is the extensional wave velocity $v_E = \sqrt{E/\rho}$ (E is Young modulus and ρ is material density). When k grows, ω in the lower branch tends to a horizontal asymptote (high frequency cut-off). The upper branch presents a low frequency cut-off typical of waveguide modes. When k grows ω approaches the asymptote $\omega = v_T k$ where v_T is the shear wave velocity in the medium. Between the upper and the lower cut-off frequencies, there is a forbidden interval of frequencies (death zone).

At small wave-numbers, the lower branch is a very good approximation to the lower mode predicted by Pochhammer - Chree dispersion equation. However, this last exact mode doesn't have a high-frequency cut-off as predicted by the GB theory. In fact when k grows the dispersion relation of this exact mode shows a tilted asymptote $\omega = v_R k$, being v_R the velocity of Rayleigh surface waves for the given material (Kolsky, 1963).

Neither the experimental results nor the detailed numerical analysis of Pochhammer-Chree exact dispersion equation, show evidence for a forbidden interval of frequencies at least in the case of cylindrical rods. Besides, the upper branch of GB theory does not describe properly the propagation of high frequency longitudinal pulses. Later numerical analysis of the exact dispersion equation (Mason, 1958) showed that, in order to describe the propagation of longitudinal pulses at high frequencies, it is necessary to take into account certain modes of fairly high order. For these modes, ω is asymptotic to $\omega = v_L k$ when k tends to infinity. Here v_L is the velocity of bulk dilatational waves in the material under consideration. These modes could explain the known experimental results about the propagation of high frequency longitudinal pulses in bars, but they are outside the scope of G.B. theory. Because it failed at high frequencies, and because the death zone that it predicted was not found, the theory had to be dismissed.

However, when purely longitudinal vibrations of relatively low frequencies are excited in tubes and rods of any cross-section, in order to study mechanical resonance, the relation between ω and k follows the lower branch of the G.B. dispersion relation, for relatively small values of k. As a consequence, the G.B. theory can be applied, as a simplified mathematical model, to the design and interpretation of mechanical resonance experiments. Indeed, as it can be applied to tubes or bars of any cross section, at low frequencies the dispersion relation of the G.B. theory has a wider scope than the exact dispersion relation of Pochhammer and Chree, because this last one was obtained for and can be applied only to, cylindrical waveguides.

Modal analysis of pulse propagation is amenable to an analytical approach, as the one intended here, if it can be done using only a few modes (two or three). So, let us retain from the theory of Giebe and Blechschmidt the idea of describing longitudinal vibrations in a cylindrical rod using only a few modes (modes of propagation, in our case) that emerge from the coupling of others more basic modes of propagation.

There is another idea that we will borrow in order to construct a simplified mathematical model for longitudinal pulse propagation. It stems from some experimental results obtained in 1948 by Mc

Skimin, working with trains of high frequency pulses in elastic rods (Mc Skimin, 1956). This author found that, when the radius to wavelength ratio is quite large, a short duration packet of dilatation waves propagates along the rod similar to a wave-packet in a pressure release fluid waveguide. Of course, mode conversion at the boundary of the elastic medium always produces an energy loss from the dilatation wave-packet. But even if we neglect mode conversion, reflection and interference of the dilatational waves at the boundary of the equivalent fluid waveguide produces a set of propagation modes each of which shows geometric dispersion. According to the results obtained by the computing group of Bell Telephone Laboratories, in connection with early delay line studies (Mc Skimin, 1956), the dispersion relation of the lowest mode of the equivalent fluid waveguide behaves very similar to the dispersion relation of the lowest mode corresponding to predominantly longitudinal waves, in the solid rod, as predicted by the equations of elasto-dynamics. This suggested the introduction of the concept of "almost-fluid waveguide" to describe with a maximum of simplicity the propagation of high frequency longitudinal waves in solid bars (Suárez-Antola, 1990). Then, in order to comply with the almost-fluid waveguide concept, we will require that the dispersion relation for the highest mode of propagation of our intended model, besides being asymptotic to $\omega = v_L k$ when k tends to infinity, must be near the dispersion relation of the first mode of the almost fluid waveguide for k big enough.

Besides the aforementioned lower extensional-surface mode and the higher shear-dilatational modes, the exact dispersion equation shows a third kind of propagation mode. The phase velocity is in this case asymptotic to v_T when k tends to infinity (asymptotic shear mode). But the group velocity of the main asymptotic shear mode, considered as a function of kR, presents a significant maximum (less than v_E) followed by a less significant minimum (higher than the minimum of the group velocity of the extensional surface mode) before approaching asymptotically to v_T (Kolsky, 1963, Fig.15). As a consequence of all this, we will require three branches to the dispersion equation obtained from our model: a lower extensional-surface branch, an asymptotically shear branch in the middle, and an upper asymptotically dilatational branch.

3. A simple mathematical model with three coupled modes of propagation.

Let us begin by considering a pure extensional mode with dispersion equation $\omega = v_E \cdot k$, and two shear modes with dispersion equations $\omega^2 = \omega^2_{C1} + v^2_T \cdot k^2$ and $\omega^2 = \omega^2_{C2} + v^2_T \cdot k^2$.
Here the cut-off frequencies are given by the equations

$$\omega_{C1} = \frac{\alpha_{T1} \cdot v_T}{R} \qquad [1a]$$

$$\omega_{C2} = \frac{\alpha_{T2} \cdot v_T}{R} \qquad [1b]$$

α_{T1} and α_{T2} are pure numbers, with α_{T1} less than α_{T2}.
These dispersion equations of shear modes are obtained after solving the equations of elasto-dynamics for an infinite waveguide of circular cross-section, but using mixed boundary conditions instead of the stress-free boundary conditions imposed by Pochhammer and Chree. Mixed boundary conditions means here that the radial component of displacement s_r and the axial component of stress σ_{rz} are both zero at the boundary of the waveguide, that is, when r = R. For the two lower and usually dominant modes we have $\alpha_{T1} = 3.83$ and $\alpha_{T2} = 7.02$ (Auld, 1973). The extensional mode also verifies mixed boundary conditions, because in this case only s_z and σ_{zz} may be different from zero. However, this mode is not an exact solution of the full elasto-dynamic equations, since it neglects the lateral contraction or dilatation due to Poisson’s effect.

As v_E is greater than v_T , we see that the curves that represent the dispersion equations of the shear modes intersect the straight line that represents the dispersion equation of the extensional mode in the ω-k plane.

Now, the coupling of intersecting modes obtained using mixed boundary conditions produce new emerging modes that can be used to describe several features of the case with stress-free boundary conditions (Achenbach, 1973; Auld, 1973).

So, let us construct a model for the propagation of longitudinal pulses, coupling three one dimensional wave equations that correspond to intersecting modes of propagation, as follows:

$$\frac{\partial^2 A}{\partial z^2} - \frac{1}{v_E^2}\frac{\partial^2 A}{\partial t^2} = K_1 \frac{\partial^2 B_1}{\partial z^2} + K_2 \frac{\partial^2 B_2}{\partial z^2} \qquad [2a]$$

$$\frac{\partial^2 B_1}{\partial z^2} - \frac{1}{v_T^2}\frac{\partial^2 B_1}{\partial t^2} - \frac{\omega_{C1}^2}{v_T^2} B_1 = K_1 \frac{\partial^2 A}{\partial z^2} \qquad [2b]$$

$$\frac{\partial^2 B_2}{\partial z^2} - \frac{1}{v_T^2}\frac{\partial^2 B_2}{\partial t^2} - \frac{\omega_{C2}^2}{v_T^2} B_2 = K_2 \frac{\partial^2 A}{\partial z^2} \qquad [2c]$$

Here K_1 and K_2 are phenomenological coupling constants that will be suitably restricted later. If $K_1 = K_2 = 0$, the coupling disappears and A(t,z) becomes the amplitude of the aforementioned pure extensional mode, while B_1(t,z) and B_2(t,z) becomes the amplitudes of certain pure shear modes.

Let us search new emerging modes of propagation substituting the following ansatz in equations [3].

$$A(t,z) = C(k) \cdot e^{j(\omega t - kz)} \qquad [3a]$$

$$B_1(t,z) = D_1(k) \cdot e^{j(\omega t - kz)} \qquad [3b]$$

$$B_2(t,z) = D_2(k) \cdot e^{j(\omega t - kz)} \qquad [3c]$$

This gives a system of linear equations for C(k), D_1(k) and D_2(k):

$$\left(\frac{\omega^2}{v_E^2} - k^2\right) C(k) = -k^2 \left(K_1 \cdot D_1(k) + K_2 \cdot D_2(k)\right) \qquad [4a]$$

$$\Delta_1(k) D_1(k) = -k^2 K_1 C(k) \qquad [4b]$$

$$\Delta_1(k) D_2(k) = -k^2 K_2 C(k) \qquad [4c]$$

By definition

$$\Delta_1(k) = \frac{\left(\omega^2 - \omega_{C1}^2\right)}{v_T^2} - k^2 \qquad [5a]$$

$$\Delta_2(k) = \frac{\left(\omega^2 - \omega_{C2}^2\right)}{v_T^2} - k^2 \qquad [5b]$$

From equations [4] we derive the dispersion equation of the proposed model:

$$\left(\frac{\omega^2}{v_E^2}-k^2\right)\cdot\Delta_1(k)\cdot\Delta_2(k)=k^4\left(K_1^2\cdot\Delta_2(k)+K_2^2\cdot\Delta_1(k)\right) \qquad [6]$$

When both coupling constants are different from zero, this dispersion equation always has three real, positive and non-intersecting solutions $\omega = \omega_\mu(k)$, defined for every real k, with $\mu = l,m,u$. If k approaches zero, the lower branch $\omega = \omega_l(k)$ approaches the dispersion equation of the uncoupled extensional mode $\omega = v_E\, k$. The middle one $\omega = \omega_m(k)$ approaches the dispersion equation of the uncoupled first shear mode and the upper branch $\omega = \omega_u(k)$ approaches the dispersion equation of the second shear mode, also when k tends to zero. But when k tends to infinity, the lower branch is asymptotic to $\omega = v_s\, k$, the middle branch is asymptotic to $\omega = v_T\, k$, and if we select K_1 and K_2 properly, the upper branch will be asymptotic to $\omega = v_L\, k$. The asymptotic phase velocities v_s, v_T and v_L can be found from equation [6]. If $v_a = \lim\limits_{k\to+\infty}\frac{\omega}{k}$, taking the limit in the dispersion equation we obtain:

$$\left(\frac{v_a^2}{v_T^2}-1\right)^2\left(\frac{v_a^2}{v_E^2}-1\right)^2=\left(K_1^2+K_2^2\right)\left(\frac{v_a^2}{v_T^2}-1\right) \qquad [7]$$

One positive root of this equation is $v_a = v_T$. But there are another two positive roots.
If $v_a \neq v_T$, we obtain

$$\left(\frac{v_a^2}{v_T^2}-1\right)\left(\frac{v_a^2}{v_E^2}-1\right)=\left(K_1^2+K_2^2\right) \qquad [8]$$

Now, let us choose $K_1^2+K_2^2$ equal to $\left(\frac{v_L^2}{v_T^2}-1\right)\left(\frac{v_L^2}{v_E^2}-1\right)$, so that $v_a = v_L$ is a second positive root. Then the third positive root $v_a = v_s$ verifies $v_s^2 = v_E^2 + v_T^2 - v_L^2$. This is the abovementioned asymptotic phase velocity of the lower branch. As will be pointed in the discussion, it should be equal to the velocity of Rayleigh waves in the material.

In Fig.(2) we sketch, in the ω-k plane, the curves $\omega = \omega_\mu(k)$ for $\mu = l,m,u$, together with the curves that correspond to the uncoupled modes used to construct the model. (Fig (2) and Fig. (3) are qualitative).

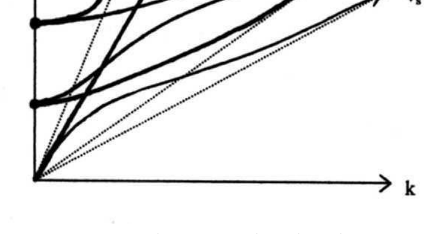

Figure 2: Upper, middle, and lower propagation modes in the ω versus k plane.

In Figure 3 we sketch the group velocities $v_{g,\mu}$ as function of the wave number.

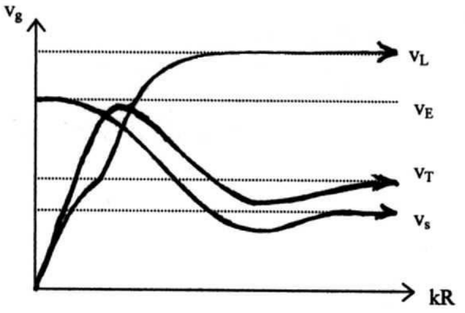

Figure3. Upper, middle, and lower propagation modes: group velocities as functions of radius and wave-number.

With a proper selection of the coupling constants the curve for $v_{g,m}$ reproduces fairly well the results of the numerical analysis of the exact dispersion equations.
Given $C_\mu(k)$ for each emerging mode μ, from equations [4b] and [4c] it is possible to calculate $D_{1,\mu}(k)$ and $D_{2,\mu}(k)$. So, the model will be complete if we establish how the mechanical energy of the wave-packet is distributed between $C_l(k)$, $C_m(k)$ and $C_u(k)$ respectively.
However, before considering this last subject, let us see if we can simplify even more our model.

4. A minimum mathematical model with two coupled modes of propagation

For a description of the geometric dispersion of longitudinal pulses in solid waveguides, the middle branch of the dispersion equation that we derived in section (C) seems to be less important than the lower and the upper branches. So, let us further simplify the model coupling the extensional mode with only one shear mode. If we put K_2=0, and if we neglect B_2 in equations [2], then only equations [2a] and [2b] remain. As a consequence, we need only equations [3a] and [3b] as the corresponding ansatz. Then, only equations [4a] and [4b] remain as means of relating C(k) and D_1(k).
Let us put $K_1 = K$, $D_1 = D$ and $\omega_{C1} = \omega_C$ (because we have only one shear mode). The dispersion equation [6] is now reduced to

$$\left(\frac{\omega^2}{v_E^2} - k^2\right) \cdot \Delta(k) = k^4 K^2 \qquad [9]$$

Here $\Delta(k)$ is given by equation [5a] with $\omega_{C1} = \omega_C$. Equation [7] for the asymptotic phase velocity reduces to

$$\left(\frac{v_a^2}{v_T^2} - 1\right)\left(\frac{v_a^2}{v_E^2} - 1\right) = K^2 \qquad [10]$$

If we establish that $v_a = v_L$ must be a positive root of equation [10], we must choose

$$K^2 = \left(\frac{v_L^2}{v_T^2} - 1\right)\left(\frac{v_L^2}{v_E^2} - 1\right) \qquad [11]$$

In that case the second positive root of equation [10] is $v_a = v_s$ and it verifies equation [8] of section (C). If $\omega = \omega_\mu(k)$ $(\mu = l,u)$ represents the two branches of the dispersion equation [9], and if we introduce the phase velocity $v_{f,\mu} = \omega_\mu(k)/k$, then for each mode of propagation the following relation is obtained between C(k) and D(k) :

$$\frac{D_\mu(k)}{C_\mu(k)} = -\frac{1}{K}\left[\left(\frac{v_{f,\mu}}{v_E}\right)^2 - 1\right] \qquad [12]$$

Note than $C_\mu(k)$ and $D_\mu(k)$ may be considered as real functions of k, and they always remain bounded. For $\mu = l$, the lower extensional-surface mode, when k approaches to zero (long wavelengths) the phase velocity $v_{f,l}$ approaches to v_E, so that $D_l(k)$ approaches to zero. For $\mu = u$, the upper shear-dilatational mode, $D_u(k)/C_u(k)$ approaches to $-\sqrt{\left(\frac{v_L^2}{v_E^2}\right)-1 \Big/ \left(\frac{v_L^2}{v_T^2}\right)-1}$ when k tends to infinity (short wavelengths). To obtain this last result we must substitute K in equation [12] by its expression as a function of v_L, v_T and v_E given by equation [11]. In order to take into due account Poisson´s effect (axial elongation (compression) appears together with lateral contraction (dilatation)), K must be considered to be positive. Thus $C_\mu(k)$ and $D_\mu(k)$ have opposite signs.

If we define non-dimensional variables $x = R\omega/v_L$ and y = k R, the dispersion equation [9] can be rewritten as follows, with $\omega_C = \alpha_T v_T/R$:

$$\left(\frac{v_L^2}{v_E^2}x^2 - y^2\right)\cdot\left(\frac{v_L^2}{v_T^2}x^2 - \alpha_T^2\right) = K^2 y^4 \qquad [13]$$

From equation [13] we can obtain x as a function of y (that is, ω as a function of k) or y as a function of x (that is, k as a function of ω). In principle we could consider two idealized situations. In one of them, the wave guide is infinite in both directions. For t = 0 we know the spatial pattern of displacements and velocities, and after that we want to follow the propagation of the elastic disturbance. The initial disturbance (pulse) can be expressed as a Fourier integral using the wave-number k. Then, to follow the propagation of the pulse we need ω as a function of k. In the other situation we have a half infinite waveguide, with an emitting transducer located at z = 0. Now we know the fields as a function of t, and we can express them as Fourier integrals using the frequencyω. To follow the propagation of the disturbance thus generated, we need k as a function ofω.

In our case, the second situation is the most interesting. However, almost all the numerical calculations done for plates, rods and other types of solid waveguides give ω as a function of k, and then give both phase and group velocities as functions of k . On the other side, for each mode of propagation the relation between ω and k is given by a strictly monotonic function (fig.2). As a consequence, once we know the modal amplitudes C_l, C_m and C_u, or the phase and group velocities as a function of k for each mode, we can univocally determine their values for any given frequencyω. So in this paper we are going to solve ω as a function of k, for purpose of comparison with the other approaches to the dispersion relation. Thus, we obtain two branches, $f_l(y)$ and $f_u(y)$, being

$$f_{\underset{(u)}{l}}(y) = \sqrt{\frac{1}{2}\left[\alpha y^2 + \beta \underset{(+)}{-} \sqrt{\left(\beta + \gamma y^2\right)^2 + \delta y^4}\right]} \qquad [14]$$

Here $\alpha = \left(\frac{v_E}{v_L}\right)^2 + \left(\frac{v_T}{v_L}\right)^2$ $\quad \beta = \alpha_T^2\left(\frac{v_T}{v_L}\right)^2$ $\quad \gamma = \left(\frac{v_T}{v_L}\right)^2 - \left(\frac{v_E}{v_L}\right)^2$ $\quad \delta = 4K^2\left(\frac{v_E}{v_L}\right)^2\left(\frac{v_T}{v_L}\right)^2$

Figure (4) shows the two branches of the simplified dispersion equation, calculated from equation [14], with $\alpha_T = 3.83$ and for the case of a steel bar: $v_T = 3230$ m/s, $v_E = 5192$ m/s and $v_L = 5900$ m/s. (Fig(4) and Fig (5) are quantitative).

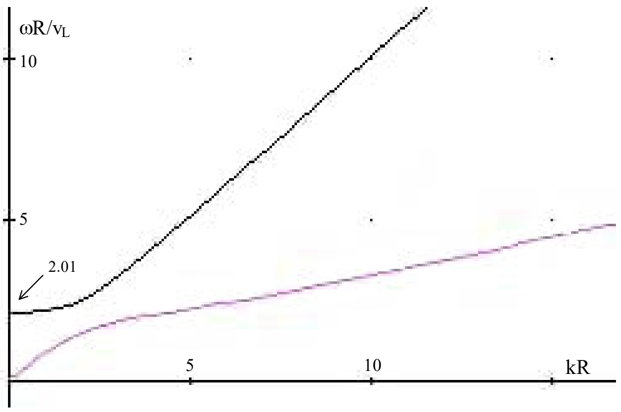

Figure 4. The dispersion relation in the ω versus kR plane: upper and lower branch.

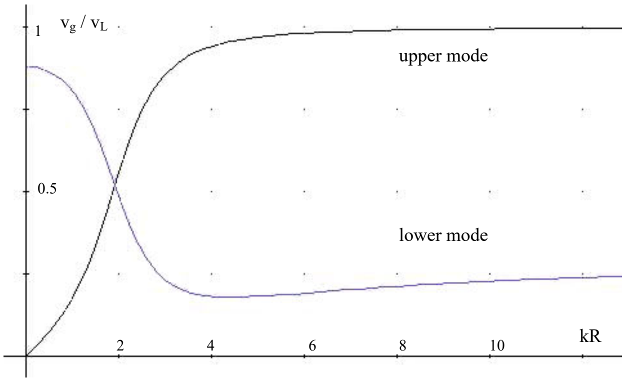

Figure 5. Upper and lower mode: group velocities as functions of radius and wave-number

Figure (5) shows the non-dimensional group velocity $v_g/v_L = \left(d/dy \right) f(y)$ is a function of y for each branch of the dispersion equation, where f(y) is $f_l(y)$ or $f_u(y)$. Because $y = kR$ we can obtain from these curves, not only how the group velocity depends of the wave-number k, but also how it depends of the bar radius R. The curve that gives the group velocity as a function of kR for the extensional-surface mode present a minimum exactly as the corresponding curve found by a detailed numerical analysis of the solutions of the Pochhammer-Chree´s exact dispersion equation. The other curve, that gives the group velocity as a function of the same independent variable for the shear-dilatational mode, begins from zero and grows towards an horizontal asymptote that corresponds to $v_g = v_L$.

5. Discussion and Conclusions

Let us begin with the distribution of the energy of the elastic disturbance amongst the propagation modes. We consider a half infinite waveguide, with circular cross-section, beginning at z = 0 where an exciting transducer is located. The transducer produces an azimuthally symmetric vectorial displacement field $\vec{s}(t,r,z)$ that propagates from z = 0 towards infinity. Now, if $\vec{s}_n(r,\omega)$

represents the amplitude of the displacement field corresponding to the Pochhammer- Chree´s mode of propagation number n, it is possible to make the following decomposition of the pulse:

$$\vec{s}(t,r,z) = \int_{-\infty}^{+\infty} \vec{s}_0(r,z,\omega) \cdot e^{j\omega t} d\omega \qquad [15a]$$

$$\vec{s}_0(r,z,\omega) = \sum_{n=1}^{\infty} a_n(\omega) \cdot e^{-jk_n(\omega)z}\, \vec{s}_n(r,\omega) \qquad [15b]$$

$$\vec{s}_0(r,0,\omega) = \sum_{n=1}^{\infty} a_n(\omega)\, \vec{s}_n(r,\omega) \qquad [16]$$

As the mode amplitudes $\vec{s}_n(r,\omega)$ are orthogonal (Auld, 1973; Achenbach, 1973) and after their normalization, we can derive a formula for $a_m(\omega)$:

$$a_m(\omega) = \int_S \vec{s}^{\,*}_m(r,\omega) \cdot \vec{s}_0(r,0,\omega) \qquad [17]$$

Here S represents the cross-section of the bar and $\vec{s}^{\,*}_m$ is the complex-conjugate displacement field of the mode of propagation number m. So $a_m(\omega)$ is given by the projection (scalar product) of $\vec{s}_0(r,0,\omega)$ onto $\mathbf{s}^*_{\mathbf{m}}$ (r, ω).

For the case of longitudinal pulses, the displacement $\vec{s}_0(r,0,\omega)$ is directed mainly parallel to the axis of the rod and it varies fairly smoothly with r.

Consider now the first mode of propagation (n = 1) corresponding to Pochhammer-Chree´s equations. The detailed numerical analysis shows that for low frequencies it corresponds fairly accurately to the extensional mode of propagation well known from the elementary theory of elastic waves in rods. But when ω increases the displacement field of this mode diminishes near the axis of the rod and increases near the surface. When the frequency is high enough, we find a kind of "skin-effect" (Rayleigh waves). As a consequence of formula [17] and the assumptions we made about of $\vec{s}_0(r,0,\omega)$, $a_1(\omega)$ vanishes at high frequencies. All the other modes present a low cut-off frequency. At frequencies lower than the lowest cut-off frequency all the energy of the pulse propagates in the extensional-Rayleigh mode with amplitude $a_1(\omega)$. However, when the frequency is increased, higher modes are excited and less and less energy remains in the first mode. As we said before, in part (B), there are some modes that at high frequencies are asymptotic to shear waves, and there are others, even higher, that are asymptotic to dilatational waves. If the (mainly longitudinal) displacement field produced by the transducer varies smoothly with the distance r , then the lowest of these asymptotic shear and asymptotic dilatational modes are mainly excited. So we can make our C_l considered as a function of ω, proportional to $a_1(\omega)$, and we could relate C_m and C_u with the amplitudes of the lowest asymptotic shear and asymptotic dilatational modes of propagation, respectively. This explains why we determined the phenomenological coupling constant K of the minimum mathematical model with two coupled modes (and why we restricted K_1 and K_2 in the model with three coupled modes) imposing the condition $\lim_{k\to+\infty} \frac{\omega_u(k)}{k} = v_L$ on the branch of the dispersion equation that corresponds to the upper mode of propagation. In fact, at high frequencies (high k) almost all the energy of the longitudinal pulse travels in the upper mode of our simplified model, with a phase velocity that approaches to v_L when k tends to infinity. In this case the asymptotic phase velocity for the lower mode, $\lim_{k\to+\infty} \frac{\omega_l(k)}{k} = v_s$, is not important. According to the proposed model, for frequencies between ω_{C1} (the cut-off frequency of the middle branch of the dispersion equation) and ω_{C2} (the cut-off frequency of the upper branch), a significant part of the energy of the pulse may travel in the shear-shear mode of propagation. The remaining energy must travel in the extensional-surface mode. For frequencies below ω_{C1} all the energy must travel in this last mode. If $\omega<\omega_{C2}$ the shear-dilatational mode will not be excited. In ultrasonic non-destructive testing with longitudinal pulses the input signal is an oscillation with the carrier frequency and with amplitude modulated to limit the duration of the pulse. When the produced elastic disturbance is measured at a given cross section of the rod, it can be described as an oscillation modulated both in amplitude and phase. The local amplitude and phase, as functions of time, can be determined using the Hilbert´s transform of the measured signal (Debnath, 1995). But this last method can be applied also to the corresponding computer results obtained from digital simulations of pulse propagation, working with ab-initio numerical methods applied to the equations of elasto-dynamics. On the other side, and in the framework of the model proposed here, the propagation of a longitudinal pulse can be described by the superposition of three wave-packets, one for each mode of propagation. Then, we encounter integrals like $\int_{-\infty}^{+\infty} C_{\mu}(\omega)\cdot e^{j(\omega t-k_{\mu}(\omega)z)}d\omega$

(μ = l, m, u) and similar integrals with $D_{1\mu}$ or $D_{2\mu}$ instead of C_{μ} . For the case of the abovementioned pulses used in non-destructive testing, it is possible to calculate these integrals in closed form fairly accurately, using asymptotic methods (Bleinstein and Hadlesman, 1986). Given an analytical description of the spectrum of the input signal and given the dispersion relation proper of a definite mode of propagation, it is possible to obtain theoretical formulae that describe fairly accurately the dispersion of the wave-packets, considered as oscillations modulated both in amplitude and in phase, at each cross-section of the rod. Some geophysical examples can be found in the book by Pilant (1979). Specific applications to longitudinal ultrasonic pulses in bars of any cross-section, using Love´s theory to approximate the extensional-surface mode at low frequencies and the almost-fluid waveguide model to approximate the shear-dilatational mode at high frequencies, can be found in Suárez Antola (1998). The detailed calculations using the system of emerging coupled modes proposed in the present paper remain to be done for different input signals. These calculations could be of interest, because the common interpretation of certain experimental results may prove to be an oversimplification of the facts. As a first example, let us consider briefly a very nice experiment done by Mayer and Neumann (1972). They used a nickel wire of 2m of length and 1.6mm of diameter. A short longitudinal pulse of carrier frequency $\omega_0 \approx$ 1.26 MHz. was produced by a coil using the magneto-strictive effect, as input signal at one end, and the emerging pulse was measured by another coil located at the other end. According to our coupled modes model, the cut-off frequency for the middle shear-shear mode is $\omega_{C1} \approx$ 3.59 MHz, so that most of the energy of the elastic disturbance should travel in the extensional-surface mode. A simple inspection of the output signal shows an oscillation with modulated amplitude and phase, whose local frequency increases with time. The usual explanation of this increase in frequency is that the phase velocity of the harmonic waves that form the pulse in the bar diminishes with frequency. So the waves with lower frequency arrive first to the end of the bar. However, a detailed analysis of how a wave-packet that travels in the extensional-surface mode is produced by superposition of travelling harmonic components shows that things are not so simple. If all the significant frequencies of the spectrum of the input signal are below the frequency that corresponds to the minimum of the group velocity in the extensional-surface mode (fig.5), then we should observe a continuous increment in the local frequency of the output signal. But if we have significant frequencies above the one corresponding to the aforementioned minimum, a more complex pattern of local frequency variation with time could be produced, in spite of the monotonic decrement of phase velocity with frequency always found in the extensional-surface mode from $\omega = 0$ to infinity.

Now, let us consider the experimental results given in Tab (1) and Fig (6) that is the relationship between the radius of a steel bar and the apparent velocity of longitudinal ultrasonic pulses.

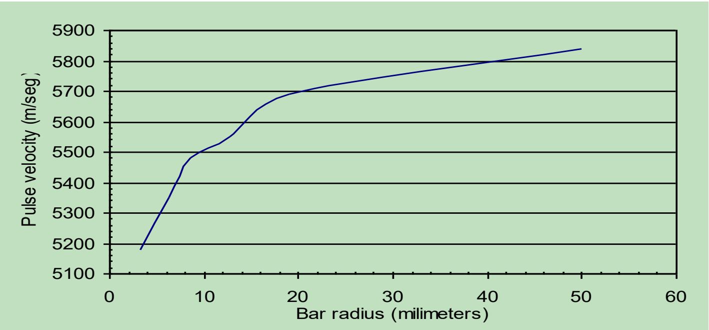

Figure 6 Experimental curve relating apparent pulse velocity with bar radius.

The pulses were produced by a probe of 50 mm of diameter, located at one flat end of the steel bar, and they were received by an equal probe located at the other flat end, so that the path length was known (approximately 2m). The receiving probe produces an output voltage that emerges from a certain weighted average of the elasto-dynamic field at the output of the bar. When this voltage overcomes for the first time a certain threshold, the corresponding instant of time appears in a digital display. The zero of time is given by the instant in which the emitting probe is excited by an electric shock to produce an ultrasonic pulse. The time dependence of the input elasto-dynamic signal can be approximated by a damped sinusoidal oscillation of carrier frequency $\omega_0 \approx 0.34$ MHz. and an exponential damping factor near $\omega_0/2\pi$.

On the contrary, the output voltage signal is a complex pattern of oscillations, modulated both in amplitude and in frequency. Nevertheless, the spectrum of the initial pulse has very strong components around the carrier frequency. Then, a naive and oversimplified explanation of the experimental results could be the following one. We suppose that the part of the output signal that is detected can be described using the method of stationary phase. We assume that the local frequency is always near to ω_0 , and that all the propagation modes of our model are excited above threshold. In that case, the apparent velocity of the pulse would be given by the fastest group velocity between those of the different modes at the given frequency ω_0 and for the given bar radius.

For example, we can use the minimum model with two coupled modes. If we have the group velocities as functions of frequency, we select the fastest one that corresponds to the given value of $\omega_0 R/v_L$. But if the group velocities are given as functions of the wave-number, as in Fig.(5), given the frequency ω_0 and the radius R, first we must calculate k (one or two different values of k for the same frequency ω_0, if ω_0 is less than or greater than the cut-off frequency ω_c , respectively). Anyhow, we obtain an estimation of the velocity of the longitudinal pulses for the given bar radius. The same procedure could be done, and with much better success, for the model with three coupled modes. But even if this procedure can in fact explain several interesting features of the experimental results obtained by Chung, things are not so easy. The length of the path of the pulse must be taken into due account. The distribution of the energy of the input pulse between the different modes of propagation as a function of bar's radius can't be neglected. As a consequence, the local frequency of the output pulse may be far from ω_0. Then, either a detailed closed form calculation using asymptotic methods applied to the model proposed in the present work, or directly an ab-initio digital simulation using the full elasto-dynamic equations for rods of different length and radius, should be done. A proper design of the digital simulation would be much aided by performing first the analytical approach.

Finally, we can establish some conclusions:

1) The analytical model that we began to construct in the present work may be considered as an improvement of the old theory of Giebe and Blechschmidt. Our model retains certain advantages of the G.B. theory (description of the longitudinal oscillations using a few modes, very good behaviour at relatively low frequencies, and applicability to bars of any cross section (after a suitable selection of the cut-off frequencies of the basic shear modes)) and avoids its main pitfalls (death interval of frequency for rods with circular cross-section, horizontal

asymptote in the ω-k plane for the lower mode, and shear wave velocity as asymptotic phase velocity for the upper mode).

2) The model has to be completed in order to obtain a simple specification of the distribution of elastic energy of the input pulse between the different modes of propagation. A part of this task was drafted in the discussion above. With this addition and with the use of the abovementioned powerful asymptotic methods, it should be possible to predict the propagation of longitudinal pulses in different solid waveguides and for different inputs. The full calculation for the models with two and three modes of propagation remains to be done, as well as the sonograms of the pulses produced by different input signals. In the case of the model with three modes, we need another relation between the phenomenological constants K_1 and K_2. This relation can be obtained imposing a condition over the maximum of the group velocity corresponding to the shear-shear mode. A more fundamental approach would be to try to obtain the simplified model from the full equations of elasto-dynamics, using a suitable version of the theory of averaged fields (Here we refer to an approach somewhat similar to the one used to obtain Darcy's law for water flow in a porous medium by a suitable averaging of the Navier-Stoke´s equations).
3) The proposed model could be adapted to take into due account loss processes in the material and then it could be used in non-destructive tests by mechanical resonance, and in a better description of the propagation of ultrasonic pulses using asymptotic methods.

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THE MEANING OF TRANSIT TIMES IN NDT OF REINFORCED CONCRETE

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Abstract

Transit times of longitudinal pulses that travel near reinforcing steel bars are usually smaller than in plain concrete. Chung first, and then Bungey, demonstrated the importance of including bar diameters in the correction factors when pulses propagating parallel and near to the bars can not be avoided. Besides the influence of geometric dispersion and mode conversion on pulse propagation, there is an effect due to a specific pattern of radial variation of longitudinal wave velocities from the surface of the embedded steel bar. Wall effects produce a region (a sheath) of concrete surrounding the steel bar, with smaller longitudinal pulse velocities in comparison with the velocity farther away from the bar. Then the acoustic energy travels trapped in a kind of composite leaky waveguide with a velocity less than the compression (P) wave velocity in steel and in concrete but greater than the extensional wave velocity in steel. A mathematical model of the propagation of a longitudinal wave-packet is constructed, taking into account both attenuation and dispersion effects. An asymptotic analysis of pulse propagation and the introduction of a threshold of detection in the receiving transducer, allows the derivation of an approximate analytical formula for transit times. Then the meaning of transit times in NDT of concrete is discussed. The abovementioned formula is used to study the effects of the reinforcing bar radius, the propagation parameters in plain concrete and in the sheath surrounding the steel bar, the path length between the transmitting and the receiving transducers, and the energy and the spectral composition of the pulse injected by the emitter in the tested body. Chung's empirical correlation between the measured pulse velocity and bar radius is reviewed and a new correlation is proposed.

Key words: ultrasonic testing of reinforced concrete, transit times of longitudinal pulses, leaky waveguide, mathematical model

1. Introduction

Ultrasonic testing of concrete is currently used for quality assessment in large concrete structures cast on site and in mass-produced prefabricated units. The intrinsic heterogeneity of concrete limits the carrier frequencies that can be used to 250 kHz and less (being 54 kHz the most common frequency) when probing distances are greater than 1 m. It usually demands two ultrasonic transducers, one for pulse emission and the other for pulse reception. In practice, probes for ultrasonic testing of concrete are not much larger (50 mm) than the customary ones for metals. The contact face of a common cylindrical probe, whose diameter is of the order of magnitude of the wavelength, or even less, radiates besides longitudinal waves, also shear and surface waves of appreciable intensity. The radiation and reception beam patterns are much less collimated than in the case of metals, although still not entirely isotropic (1). In any case, the absence of any significant directional effect in the probes, as well as the multiple scattering of waves inside concrete, makes it possible, (in principle) to couple directly any two points on the surface of the specimen being tested. The fastest ultrasonic signal received in this way is then always a direct longitudinal wave-pulse. This is followed by shear and surface wave-pulses, and by reflected longitudinal waves. For quality tests on concrete the longitudinal acoustic velocity is determined measuring the distance between probes and the time that the direct longitudinal pulse takes to go from the emitting to the receiving probe. A higher velocity usually means a better concrete quality (strength, durability and dimensional stability). Dry cracks in concrete members may be detected and their depth and inclination assessed by careful measuring time-versus distance relationships for longitudinal pulses. Cavities and micro-crack fields can be detected under suitable conditions.

In all this measurements, reinforcement, if present, should be avoided. Indeed, considerably uncertainty is introduced by the higher velocity of pulses in steel and by possible compaction shortcomings in heavily reinforced regions. If the reinforcing bars run in a direction at right angles to the pulse path and the sum of bar diameters is small in relation to path length, the effect is generally not significant. But if the reinforcing bars lie along or parallel to the path of the pulse, and their diameter is greater than 10 mmm, the effect may be very significant. It is not only the measurements taken along the reinforcing bars that are affected, but also those in the neighbourhood of the bars. The first pulse taking an indirect path connected with the reinforcing bar may reach the receiving probe before that going along the direct path through bulk concrete, resulting in a shorter apparent transit time. Now, a relatively small difference in pulse velocity usually corresponds to a relatively large difference in the quality of concrete. So, if reinforcement cannot be avoided, it may be necessary to make some corrections in order to estimate the true longitudinal pulse velocity in bulk concrete. The purpose of the present work is to give a theoretical discussion of the relationship between the propagation time of longitudinal pulses and the parameters of reinforced concrete, of the ultrasonic transducers (emission and reception properties) and the detection threshold of the equipment. The intention is to improve and enlarge, with the help of fairly simple mathematical models, the discussion already begun in a previous paper (2). As was noted there, even if enough spatial symmetry is assumed, the problem of pulse propagation in steel bars embedded in concrete is fairly difficult to solve using directly an ab-initio numerical analysis of the equations of elastodynamics. So, a simplified analytical approach may be of interest, both for the design of realistic digital simulations and for the interpretation of the experimental results obtained with non-destructive testing (ultrasonic) equipments.

RILEM recommendations commonly used in ultrasonic testing of structures for making corrections in pulse velocity measurements, when the reinforcement influence cannot be avoided, are briefly reviewed. An extension of this approach to estimate the transit times of longitudinal pulses in a reinforced concrete with two layers of different quality surrounding the steel bar is suggested.

Some significant limitations of the ray theory of wave propagation that serves as framework for the abovementioned recommendations are considered. The results of Chung's experiments are briefly summarized. Then, a simplified model called "almost-fluid leaky waveguide model", is used to discuss the dispersive propagation of longitudinal pulses in reinforced concrete. Powerful asymptotic methods are applied to give a better description of wave-packet propagation, beginning with an analysis of the elastic precursor. A new and theoretically grounded correlation is proposed between velocity, bar radius, and other parameters of practical interest in ultrasonic NDT of concrete in structures.

2. The standard corrections and some of their limitations

The following correction is recommended by RILEM and British Standards (3) for the situation depicted in fig 1:

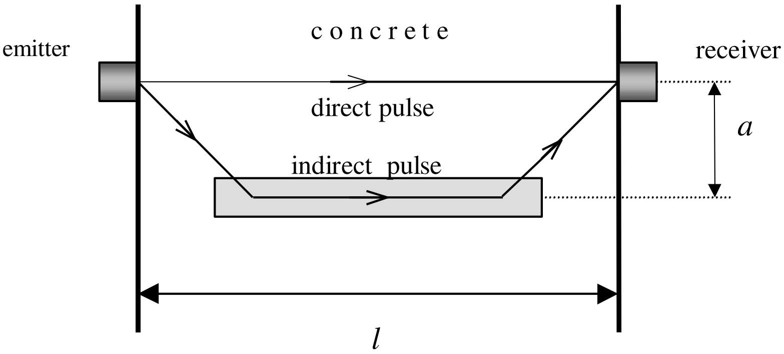

Figure 1: Direct and direct paths for ultrasonic pulses in concrete.

If t is the measured travel time of the pulse, l the apparent path length, V_S the pulse velocity in steel (which may be from 1.4 to 1.7 higher than the pulse velocity in plain concrete) and a is the distance shown in the figure, then the travel time of an indirect pulse, arriving first, and given by elementary acoustic ray theory is:

$$t = \frac{l}{V_s} + 2.a.\frac{\sqrt{V^2{}_s - V_c^2}}{V_s.V_c} \qquad [1]$$

According to BS 4408 pt.5 and from [1], it follows that the true pulse velocity in bulk concrete, V_c is given by $V_c = \frac{2aV_s}{\sqrt{4a^2 + (tV_s - l)^2}}$, while the apparent pulse velocity in bulk concrete would be $V_{c,app} \equiv \frac{l}{t}$. This correction should be applied if $V_c \le V_s$ and if a/l is less than the critical value: $\frac{1}{2}\sqrt{\frac{V_s - V_c}{V_s + V_c}} = \left(\frac{a}{l}\right)_{critical}$ [2]

The effect of the reinforcement disappears if a/l is greater than this critical value, because in this case the direct pulse arrives first. As a consequence, the "apparent velocity" l/t is now the true velocity in plain concrete.

The aforementioned correction formula is based in the acoustic ray theory of the old refraction method of the seismologists. In this case we have an elastic layer of thickness a over an elastic half-infinite bed. P waves are produced by an emitter at the upper face of the layer, and the waves are detected by a receiver, also located at the upper face. If the bulk P wave velocity in the bed is higher than the bulk P wave velocity in the layer (concrete), an indirect P wave arrives first after travelling totally reflected at the interface. All the other indirect P waves arrive later.

However, the concrete surrounding the bar is often of different quality that plain concrete (in the adjacent layer, P waves travel slower than in the bulk of the material). By a direct extension of the acoustic ray theory to two layers in a half infinite bed (4) well known to geophysicists (Fig.2), the following generalization of the travel time of indirect ultrasonic pulses is obtained for reinforced concrete:

$$t = \frac{l}{V_s} + 2.a_1.\frac{\sqrt{V^2{}_s - V^2{}_{c1}}}{V_s.V_{c1}} + 2.a_2.\frac{\sqrt{V^2{}_s - V^2{}_{c2}}}{V_s.V_{c2}} \qquad [3]$$

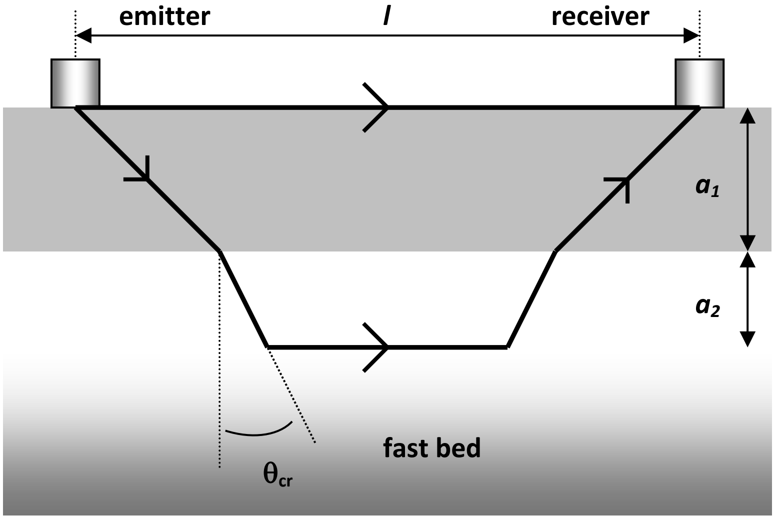

Figure 2: The refraction method for two layers on a fast half-infinite bed

Here a_1 is the depth of the upper layer with higher P wave velocity V_{c1}, while a_2 and V_{c2} are the corresponding parameters for the layer adjacent to the steel bed, with slower P velocity. The travel time of the direct pulse is: $t = {}^{l}\!/_{V_{c1}}$ The indirect pulse arrives first if ($a = a_1 + a_2$):

$$\frac{a}{l} \le \frac{1}{2}\sqrt{\frac{V_s - V_{c1}}{V_s + V_{c1}}} - \frac{a_2}{l}\left(\sqrt{\frac{V^2{}_s - V^2{}_{c2}}{V^2{}_s + V^2{}_{c2}}} - 1\right) = (\frac{a}{l})_{critical} \qquad [4]$$

In this formula V_{c1} corresponds to V_c in formula [2]. So, the critical ratio $({}^{a}\!/_{l})_{critical}$ is less if there is a layer of relatively low velocity concrete between plain concrete and steel. Formula [4] could give a more realistic but less conservative estimation of the critical ratio of influence of steel on longitudinal pulse velocity measurements in reinforced concrete. Nevertheless, to represent reinforcement bars as equivalent to a half infinite steel bed, is indeed a very coarse simplification of the actual situation, because the radius of the bar is fairly small and the wavelengths are fairly large compared with the other lengths involved in pulse propagation.

Chung (5) has shown that for pulses travelling in the direction of the axis of the reinforcing bars through a steel-concrete medium, the effective pulse velocity V_e is less than the above mentioned value V_s for P waves in bulk steel and it is influenced by bar's diameter according to the approximate empirical formula:

$$V_e = 5.90 - \frac{5.2}{R}.(5.90 - V_c) \qquad [5]$$

Here R is bar radius (in mm and greater than 5.2) and V_c is the true velocity in bulk concrete (in km/sec.). Chung's experiments were done for six concretes mixes (of different proportions of water, cement, sand and coarse aggregates) but always with the same transducers of a carrier frequency of 54 kHz and for fixed path lengths (750 mm) and transverse dimensions (150 by 150 mm).

As a consequence of Chung's findings, it seems that some corrected V_e must be used instead of V_S in formula [1]. In BS 4408 pt.5 a fixed value of 5.5 km/sec is suggested for V_S. This figure is a fixed average between longitudinal bulk wave velocity (5.9 km/sec) and extensional (bar) wave velocity (5.2 km/sec) in steel. As such, it doesn't take into account neither bar diameter nor a possible effect due to the pulse spectrum and path length that may be reasonably expected due to geometric dispersion and spatial attenuation of the longitudinal wave-packet (6). Further experimental work about the influence of reinforcement and a critical discussion of the proposed corrections was given by Bungey (7). However, even now there is controversy about the meaning and the feasibility of these corrections. It is the pulse travelling in the sheath of concrete surrounding the bar which is being measured, rather than the pulse travelling through the bulk of the material (which is almost always the goal of the measurements). Besides, the degree of adherence between steel and concrete must be taken into account when considering the influence of reinforcement bars in ultrasonic measurements. Let us try another approach to these problems, leaving geometric acoustics aside and considering waveguide effects in reinforced concrete members.

3. The almost-fluid waveguide approach and a theoretical foundation to Chung´s empirical formula

In a steel bar in air, as the wave train progresses along the bar, disturbances along the boundary are produced by total reflection and mode conversion at the steel-air interface, so as to reduce there the stress to zero. At the receiving probe a complex time signal is obtained corresponding to a longitudinal wave packet arriving first followed by several trailing pulses. Experimental results obtained by Mc Skimin in the Bell Laboratories during a research in acoustic delay lines, suggest that, as a first approximation when the wavelength is small relative to bar radius, the dilatational component may be considered to be guided along the bar much as in the case of a fluid wave-guide that releases pressure at its boundary (8). Due to interference from waves produced at interfaces, both the axial and the radial displacement fields, as functions of the distance from bar axis, show small oscillations superposed on a fairly smooth trend. If we make an average of these fields over a suitable chosen (and movable) plane region, we obtain a smoothed axial field that seems to obey very approximately a leaky classical wave equation. In the solid wave guide case energy is continually being drained off through mode conversion processes at the boundary. Nevertheless, at any cross section of the bar, the leading pulse can be reconstructed from the Fourier spectrum of the longitudinal pulse injected by the emitter using suitably defined propagation parameters and working with a properly defined average dilatational wave field. The displacement field imposed by the emitting transducer excites mainly the first mode of the almost fluid waveguide, and well known formulae for phase and group velocity as functions of frequency can be used as a first approximation to the propagation process. (The fluid waveguide concept was further developed and used to study some problems related with the influence of geometric dispersion in ultrasonic testing with longitudinal pulses in (9) and (10)).

In a steel bar immersed in concrete and well bonded to it, the P wave velocity in concrete and the S (shear) wave velocity in concrete are less than the P and S velocities in steel. So, contrary to the bar in air case, now there is no possibility of total reflection when a wave comes to the steel-concrete interface from the steel side. Then a dilatational pulse that travels along an embedded reinforcement bar is continuously loosing energy towards the adjacent concrete. Once the waves go through the steel-concrete interface towards the concrete, they scatter, reflect and convert modes many times at the interfaces between the aggregates and the cement paste. Part of this energy comes back and now part of it can be trapped by total reflection in the concrete layer adjacent to the steel bar. This trapping is enhanced when a sheath of concrete surrounding the steel bar has lower propagation velocities than the bulk of the material, due to wall effects during filling and compaction. A certain dilatational wave field is thus produced, in the concrete surrounding the bar and in the steel, inside the bar, moving along the bar axis and tending fairly quickly to zero as the distance from the bar axis grows. A suggestion was made (2) to model the propagation of the leading longitudinal pulse (whose travel time is measured in non-destructive testing) using an equivalent leaky waveguide, coaxial with the steel bar, but with an effective radius R_e, greater than bar radius R. To simplify the analysis, let us suppose for a moment that the waveguide can be treated as almost-fluid. Then, for the first propagation mode we can apply the following formula for the wave number k as a function of frequency ω:

$$k(\omega) = \frac{\sqrt{\omega^2 - \omega_{MS}{}^2(\omega)}}{V_S} \qquad [6]$$

What may be called called Mc Skimin frequency function $\omega_{MS}(\omega)$ is given by the following asymptotic expression that was introduced in reference (9):

$$\omega_{MS}(\omega) \approx \omega_C \cdot \left(1 + \sum_{n=1}^{\infty} c_n \cdot \left(\frac{\omega_C}{\omega}\right)^n\right) \qquad [7]$$

The asymptotic cut-off frequency of the mode is given by: $\omega_C = \frac{a_0 \cdot V_S}{R_e}$

[8]

Here a_0 is the first zero of the Bessel function of the first kind and order zero $J_0(z)$, and may be approximated by 2.40.

The group velocity will be: $$V_g = V_S \cdot \frac{\sqrt{1 - \frac{\omega^2{}_{MS}(\omega)}{\omega^2}}}{\left(1 - \frac{\partial^2 \omega_{MS}(\omega)}{\partial \omega^2}\right)} \qquad [9]$$

If the measured pulse velocity were the group velocity, formulae [7] to [9] taken together would be a theoretical counterpart to the empirical formula due to Chung, but with the equivalent bar radius R_e instead of the true bar radius R. Now, a relation between these two parameters must be found. In order to find it, let us work in the high frequency limit, where $\omega_{MS} \approx \omega_C$ is taken to be constant. The group velocity then simplifies to the well known waveguide formula: $V_g = V_S \cdot \sqrt{1 - \frac{\omega_C{}^2}{\omega^2}}$ [10]

It decreases when R_e decreases, being equal to the longitudinal velocity in the sheath of concrete V'_C for a critical value $\tilde{R}_e$ of the radius of the equivalent waveguide. From [8] and [10] it follows that:

$$\tilde{R}_e = \frac{\frac{a_0 \cdot V_S}{\omega}}{\sqrt{1 - \left(\frac{V'_C}{V_S}\right)^2}} \qquad [11]$$

If R_e is greater than $\tilde{R}_e$, the first pulse that arrives to the receiving transducer would be the leaky waveguide pulse. If R_e is smaller than $\tilde{R}_e$, the first pulse that arrives to the receiving probe would be the direct pulse that travels with velocity V'_C. Then, the effective velocity V_e of longitudinal pulses in reinforced concrete would be given by equations [8] and [10] if $R_e \geq \tilde{R}_e$, and by $V_e = V'_C$ if $R_e < \tilde{R}_e$.

But the formula for group velocity in the leaky waveguide can be approximated by the linear relationship: $V_g \approx a + \frac{b}{R_e}$ When R_e tends to ∞ then V_g must tend to V_S, so that $a = V_S$. When $R_e = \tilde{R}_e$ then V_g must be equal to V'_C, so that $b = -\tilde{R}_e \cdot (V_S - V'_C)$.

Thus the following Chung-like formula is derived:

$$V_e \approx V_g \approx V_S - (V_S - V'_C) \cdot \frac{\tilde{R}_e}{R_e} \qquad [12]$$

Chung's experimental formula can be re-written in this way:

$$V_e \approx V_S - (V_S - V_C) \cdot \frac{R_0}{R} \qquad [13]$$

Here V_C is the longitudinal wave velocity in the bulk of concrete, while $V'_C \approx \kappa \cdot V_C$ is the longitudinal wave velocity in the layer that surrounds the bar. The parameter κ, usually less than one, gives the ratio of velocities. If in a first approximation κ is taken equal to one, then a direct comparison between [12] and [13] gives: $\frac{R_0}{R} = \frac{\tilde{R}_e}{R_e}$ [14]

Following an argument already given in (2), it is possible to show that:

$$R_0 = \frac{a_0 \cdot V_S}{\omega} \qquad [15]$$

From this last equation and from [11] and [14] it follows that the radius of the equivalent waveguide is related with bar radius by:

$$R_e = \frac{R}{\sqrt{1 - \left(\frac{V'_C}{V_S}\right)^2}} \qquad [16]$$

The same equation relates $\tilde{R}_e$ with R_0. If the steel bar is in void ($V'_C = 0$), we obtain $R_e = R$ as it should be. If V'_C approaches to V_S, R_e tends to infinity so we will have homogeneous dilatation plane waves moving without geometric dispersion in an unbounded steel medium, also as it should be.

4. Asymptotic expansions for propagating pulses and elastic precursors

Now let us put an emitter to inject at a certain cross-section of the waveguide, of axial coordinate $z = 0$, a propagating wave packet. Let us suppose that the axial displacement of the injected longitudinal pulse may be represented as a damped sinusoid:

$$u(t, z = 0) = A_0 \cdot e^{-\mu_0 \cdot t} \sin(\omega_0 \cdot t) \cdot H(t) \qquad [17]$$

Here ω_0 is the carrier frequency and $H(t)$ is Heavside's unit step function (equal to 1 if $t > 0$ and to 0 if $t < 0$). If the pulse is a short one, not more than 5 or 6 cycles, as in ultrasonic non-destructive testing of concrete, the damping parameter μ_0 is of the order of $\omega_0/2\pi$. The amplitude spectrum $A(\omega)$ and phase $\phi(\omega)$ of this pulse are given by:

$$A(\omega) = \frac{1}{2\pi} \frac{\omega_0 . A_0}{\sqrt{\left(\mu_0{}^2 + \omega_0^2 - \omega^2\right)^2 + 4\mu_0{}^2 . \omega^2}} \qquad [18a]$$

$$\tan \phi(\omega) = \frac{-2 \cdot \mu_0 \cdot \omega}{\omega^2 - \left(\omega^2{}_0 + \mu^2{}_0\right)} \qquad [18b]$$

So, even if we have spectral components of every possible frequency, their amplitudes tend to zero as $1/\omega^2$ when ω grows to infinity.

A receiver is located in another cross-section. This probe produces a scalar (voltage) time signal after making a certain weighted average of the elastodynamic field. So, it will be enough to consider the propagation of a representative scalar field (for example, the average axial displacement) given by:

$$u(t, z) = \int_{-\infty}^{+\infty} A(\omega) . e^{i\phi(\omega)} . e^{-z\alpha(\omega)} . e^{i(\omega \cdot t - z \cdot (k(\omega) + \beta(\omega)))} d\omega \qquad [19]$$

Here $\alpha(\omega)$ is an attenuation coefficient related with the dilatational energy drainage off the equivalent waveguide, by mode conversion and by leakages towards the bulk of concrete. (The attenuation due to energy dissipation in concrete at testing frequencies, related with hereditary effects (11) or coupled elastic and thermal effects (12), is much less important than the attenuation due to heterogeneities in the material (6)). If there is energy dissipation at a given frequency, given by $\alpha(\omega)$, there will be accompanying dispersion given by the wave-number $\beta(\omega)$. These functions of frequency are related by the Hilbert Transform or, if $\alpha(\omega)$ is symmetric and $\beta(\omega)$ anti-symmetric, by the Kramers-Krönig relations (13). Nevertheless, in what follows, and as a first approximation to simplify the analysis, we will neglect the effect of $\beta(\omega)$ relative to $k(\omega)$ in the description of the propagated disturbance.

When t is big enough, the method of stationary phase (14) can be applied to the evaluation of the integral [19]. For a given t and z, the main contribution to the integral comes from the frequencies $\omega_e(t, z)$ that verify:

$$\frac{\partial k}{\partial \omega} = \frac{t}{z} \qquad [20]$$

If [20] does not have real roots, $u(t,z)$is negligible. In our case, from [9] and [20] it follows that:

$$\frac{z}{V_S \cdot t} = \frac{\sqrt{1 - \frac{\omega^2{}_{MS}(\omega_e)}{\omega_e{}^2}}}{\left(1 - \frac{\partial^2 \omega_{MS}(\omega_e)}{\partial \omega^2}\right)} \qquad [21]$$

If there is only one solution to equation [21], and $\frac{\partial^2 k(\omega_e)}{\partial \omega^2} \neq 0$ is not too small, the scalar field [19] can be approximated (14), (15), when $\omega_C \cdot t$ is large enough, by

$$u(t,z) \approx \frac{2e^{-z.\alpha(\omega)}.A(\omega_c)}{\left[\frac{1}{2\pi}.\left|\frac{\partial^2 k(\omega_c)}{\partial \omega^2}\right|.z\right]^{\frac{1}{2}}}.\cos\left(\omega_e.t - k(\omega_e).z + \phi(\omega_e) + s.\frac{\pi}{4}\right) \qquad [22]$$

Here s is the sign of $\frac{\partial^2 k(\omega_e)}{\partial \omega^2}$. Formula [22] is a local approximation by a harmonic wave of frequency $\omega_e(z/V_S \cdot t)$and wave number $k(\omega_e(z/V_S \cdot t))$. But when $z/_{t \cdot V_S}$ varies, $\omega_e$changes as well (according to formula [21]), so that we obtain a wave modulated in amplitude, frequency and phase. When the dimensionless number $z/_{t \cdot V_S}$ approaches to 1 from below, $\omega_e(z/V_S \cdot t)$ tends to infinity, ω_{MS} tends to ω_C and $\frac{\partial^2 k(\omega_e)}{\partial \omega^2}$ as well as all the higher order derivatives tends to zero. So, we can expect that at and near the wave-front of the propagated ultrasonic disturbance formula [22] is not valid. However, the relation between k and ω as well as the spectrum of the injected pulse, extended to the complex plane are such that the method of uniform asymptotic expansions developed in (16) can be applied. Neglecting the attenuation related with $\alpha(\omega)$the following simplified asymptotic formula is obtained, as a coarse approximation for the wave-front of the pulse, when $\left(1 - z/_{t \cdot V_S}\right)$ and $\omega_C \cdot t \cdot \left(1 - z/_{t \cdot V_S}\right)$, are both small:

$$u(t,z) \approx A_0 \cdot \frac{\omega_0}{2\pi} \cdot \sqrt{\frac{t - \frac{z}{V_S}}{\left(\frac{z \cdot \omega^2{}_C}{2 \cdot V_S} + 2 \cdot \mu_0\right)}} \cdot J_1\left(2 \cdot \sqrt{\left(\frac{z \cdot \omega^2{}_C}{2 \cdot V_S} + 2 \cdot \mu_0\right)} \cdot \sqrt{t - \frac{z}{V_S}}\right) \qquad [23]$$

Here $J_1(x)$ is the Bessel function of first kind and order one, and $t \geq z/_{V_S}$. When $t < z/_{V_S}$, all the fields are zero, because the acceleration front has not arrived yet to the corresponding points z of the waveguide axis. Formula [23], albeit different, is closely related with Sommerfeld´s formula for the propagation of precursors in optical dispersive media and has a similar meaning: immediately upon its arrival at the cross-section located in z, the initial amplitude is very small compared with A_0, and the initial frequency of oscillation is very high compared with the emitter nominal frequency. But the amplitude (due to the factor $\sqrt{t-\frac{z}{V_S}}$) and the period of oscillation (due to the location of the zeroes of the Bessel function), increase with increasing $t-\frac{z}{V_S}$

5. Conclusions: detection thresholds and the meaning of the measured transit times of longitudinal pulses.

The leading edge of the first dilatational pulse that is measured by the receiving probe is occupied by the highest frequencies that give amplitudes above a certain measurement threshold (1). If these frequencies are big enough, we can expect to be able to discuss the measured transit time between emitter and receiver using the almost-fluid approach. Let $\omega_{e,u}$ be the biggest frequency whose amplitude is over the detection threshold. If the asymptotic approximation [22], that represents the propagated disturbance far enough from the acceleration front of the pulse, describes the local behaviour that corresponds to the threshold frequency, and assuming that the wave-number is given by [6], at least with enough approximation, then [21] is verified. It follows that for a certain distance l between the receiving and the emitter transducers, the first signal detected would be registered in an instant t such that:

$$t = V_S \cdot l \cdot \frac{\left(1-\dfrac{\partial^2 \omega_{MS}(\omega_e)}{\partial \omega^2}\right)}{\sqrt{1-\dfrac{\omega^2{}_{MS}(\omega_e)}{\omega_e{}^2}}} \qquad [24]$$

This would give us the apparent dilatational pulse velocity l/t measured with a propagating wave-packet of a given spectrum of Fourier components and with a receiver with a certain detection threshold. The effective longitudinal pulse velocity V_e would be given by formula [9], but for $\omega = \omega_{e,u}$ instead of the carrier frequency ω_0 of the emitted pulse. Furthermore, we must use $\omega_{e,u}$ instead of ω_0 in formula [15] for the radius R_0 of the bar such that for it $V_e = V_C$.

In principle, the spectrum of the incoming pulse has component frequencies as big as we want. As a consequence, a certain fraction of the pulse, represented by equation [23], will always travel with a velocity very near to V_S. But this fraction can be detected, and its corresponding travel time can be measured, only if their amplitudes are above the detection threshold of the ultrasonic equipment. This is the case when the length l of the trajectory of the pulse is of the same order of magnitude or less than the radius R_e of the equivalent waveguide: here we do not have geometric dispersion. But when l increases, this high frequency part of the wave-packet will remain under the threshold of detection and the measured transit time will correspond to the travel time of the highest detectable frequency $\omega_{e,u}$. This threshold frequency will depend of l, of the amplitude spectrum of the emitted ultrasonic pulse, of the signal to noise characteristics of the time measuring equipment and of the propagation distortions from the emitter to the receiver described by formula [22]. The fluid waveguide model predicts that $\omega_{e,u}$ should grow if l decreases, if ω_0 (the carrier frequency) increases, and if the total energy of the incoming pulse increases (for the same relative amplitudes). Now, considering the generalization of Chung´s formula given by equation [13], from the above remarks and from equation [15] it follows that in this equation the parameter R_o should decrease if l decreases, or if ω_0 increases, or if the total energy of the pulse increases. As was already noticed in (2), from Chung´s experimental value for R_o (0.0052m) and using [15], it results $\omega_{e,u} \approx 14.160/0.0052 = 2.723$ MHz. The carrier frequency is $\omega_0 = 6.28*540$ kHz $= 0.3391$ MHz so that in this case $\omega_{e,u}$ would be nearly eight times ω_0.

Using the concept of an equivalent almost-fluid waveguide we have thus explained some known aspects of the geometric dispersion of longitudinal pulses in reinforcement bars.

This almost-fluid approximation is very useful to study the effects of the geometric dispersion when the wavelengths are small relative to the smallest dimension of the solid body perpendicular to the direction of pulse propagation. But for the low frequencies used in ultrasonic testing of concrete the wavelengths λ are not small in comparison with bar radius R (for 54 kHz in steel, $\lambda = 0.11$ m). So, as was also stressed in reference (2), even if the spectra of short ultrasonic pulses of 54 kHz of carrier frequency have significant components at higher frequencies (well above 150 kHz), a less simplified approach must be used to study the travel time of the leading dilatational pulse.

As shown in (17) and (18), a model with two or three coupled propagation modes may be enough to describe the main features of mechanical vibrations and pulse propagation in a bar of any uniform cross-section, immersed in air. If $R_e \cdot \omega / V_S$ is not large enough, the almost-fluid model cannot be applied to reinforced concrete and a more realistic model of an equivalent solid waveguide must be constructed. The simplest solid waveguide model can be obtained coupling a purely extensional mode with a suitable shear mode. This, as will be developed elsewhere, gives two emerging modes. When the dimensionless number $R_e \cdot \omega / V_S$ increases, the upper mode should approach the first dilatational mode of the almost fluid waveguide. At its turn, it seems that the lower mode approaches a kind of interface concrete-steel wave propagation mode, which is very slow and it is less and less excited by the emitting transducer as $R_e \cdot \omega / V_S$ increases. All this deserve further experimental and theoretical research, including digital simulations using suitable numerical codes. Several qualitative and quantitative predictions that stem from the results obtained in sections 3 and 4 are also amenable to experimental or digital simulation testing.

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